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Osaka J. Math.
44 (2007), 147–158

ORDINARY INDUCTION FROM A SUBGROUP AND FINITE GROUP BLOCK THEORY

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(Received January 16, 2006, revised April 21, 2006)

Abstract

The first step in the fundamental Clifford Theoretic Approach to General Block Theory of Finite Groups reduces to: H is a subgroup of the finite group G and b is a block of H such that $b({}^g b) = 0$ for all $g \in G - H$. We extend basic results of several authors in this situation and place these results into current categorical and character theoretic equivalences frameworks.

1. Introduction and statements of results

Let G be a finite group, let p be a prime integer and let $(\mathcal{O}, \mathcal{K}, k)$ be a p-modular system for G that is "large enough" for all subgroups of G (i.e., $\mathcal{O}$ is a complete discrete valuation ring, $k = \mathcal{O}/J(\mathcal{O})$ is an algebraically closed field of characteristic p and the field of fractions $\mathcal{K}$ of $\mathcal{O}$ is of characteristic zero and is a splitting field for all subgroups of G).

Let N be a normal subgroup of G and let γ be a block (a primitive) idempotent of $Z(\mathcal{O}N)$. Set $H = \mathrm{Stab}_G(\gamma)$ so that $N \leq H \leq G$. Also let $Bl(\mathcal{O}H|\gamma)$ and $Bl(\mathcal{O}G|\gamma)$ denote the set of blocks of $\mathcal{O}H$ and $\mathcal{O}G$ that cover γ, resp. Then it is well-known that if $b \in Bl(\mathcal{O}H|\gamma)$, then $b({}^g b) = 0$ for all $g \in G - H$ and the trace map from H to G, Tr_H^G, induces a bijection $\mathrm{Tr}_H^G\colon Bl(\mathcal{O}H|\gamma) \to Bl(\mathcal{O}G|\gamma)$ such that corresponding blocks are "equivalent." This basic analysis pioneered by P. Fong and W. Reynolds (cf. [5, V, Theorem 2.5]) is the first step in the fundamental Clifford theoretic approach to general block theory: the reduction to the case of a stable block of a normal subgroup.

Consider the more general situation: (P) H is a subgroup of G and e is an idempotent of $Z(\mathcal{O}H)$ is such that $e({}^g e) = 0$ for all $g \in G - H$.

Note that if β is an idempotent of $Z(\mathcal{O}H)$ such that $e\beta = \beta$, then $\beta({}^g \beta) = 0$ for all $g \in G - H$.

Fundamental contributions to this context appear in [9, Theorem 1] and in [11, Theorem 1].

The purpose of this paper is to put the significant results of [9, Theorem 1] and [11, Theorem 1] into current categorical and character theoretic equivalences context and to extend these basic results in this context.

2000 Mathematics Subject Classification. 20C20.

It is also well-known that if H is a subgroup of G and if $\chi \in \mathrm{Irr}_{\mathcal{K}}(H)$ is such that $\mathrm{Ind}_H^G(\chi) \in \mathrm{Irr}_{\mathcal{K}}(G)$ and if $e_\chi = (\chi(1)/|H|)\big(\sum_{h\in H} \chi(h^{-1})h\big)$ denotes the primitive idempotent of $Z(\mathcal{K}H)$ associated to χ, then $e_\chi({}^g e_\chi) = 0$ for all $g \in G - H$ and $\mathrm{Tr}_H^G(e_\chi)$ is the primitive idempotent of $Z(\mathcal{K}G)$ associated to $\mathrm{Ind}_H^G(\chi)$ (cf. Corollary 1.5).

In this article, we shall generally follow the (standard) notation and terminology of [5] and [10].

All rings have identities and are Noetherian and all modules over a ring are unitary and finitely generated left modules. If R is a ring, then R-mod will denote the category of left R-modules and R^0 denotes the ring opposite to R.

The required proofs of the following main results will be presented in Section 3. Section 2 contains basic results that are needed in our proofs. We shall assume that H is a subgroup of the finite group G in the remainder of this section and we shall let T be a left transversal of H in G with $1 \in T$. Thus $G = \bigcup_{t\in T} tH$ is disjoint.

For our first three results, $\mathcal{O}$ will denote a commutative Noetherian ring.

Our first two results are well-known and easy to prove (cf. [10, Sections 9 and 16]).

Lemma 1.1. *Let B be a unitary $\mathcal{O}$-algebra that is an interior H-algebra* (*as in* [10, Section 16]). *Then*:
(a)

$$\mathrm{Ind}_H^G(B) = \mathcal{O}G \otimes_{\mathcal{O}H} B \otimes_{\mathcal{O}H} \mathcal{O}G = \bigoplus_{s,t\in T} (s(\mathcal{O}H) \otimes_{\mathcal{O}H} B \otimes_{\mathcal{O}H} (\mathcal{O}H)t^{-1})$$
$$\cong \bigoplus_{s,t\in T} (s \otimes_{\mathcal{O}} B \otimes_{\otimes} t^{-1})$$

is a unitary interior G-algebra with $1_{\mathrm{Ind}_H^G(B)} = \sum_{t\in T}(t \otimes_{\mathcal{O}} 1_B \otimes_{\mathcal{O}} t^{-1})$ and with $\phi\colon G \to \mathrm{Ind}_H^G(B)^\times$ such that $g \mapsto \sum_{t\in T}(gt \otimes_{\mathcal{O}} 1_B \otimes_{\mathcal{O}} t^{-1})$ for all $g \in G$. Moreover $\{t \otimes_{\mathcal{O}} 1_B \otimes_{\mathcal{O}} t^{-1} \mid t \in T\}$ is a set of orthogonal idempotents of $\mathrm{Ind}_H^G(B)$; and
(b) *The map $\alpha\colon Z(B) \to Z\big(\mathrm{Ind}_H^G(B)\big)$ such that $z \mapsto \sum_{t\in T}(t \otimes_{\mathcal{O}} z \otimes_{\mathcal{O}} t^{-1})$ for all $z \in Z(B)$ is an $\mathcal{O}$-algebra isomorphism.*

Proposition 1.2. *Let e be an idempotent of $Z(\mathcal{O}H)$ such that $e({}^g e) = 0$ for all $g \in G - H$ and set $E = \mathrm{Tr}_H^G(e) = \sum_{t\in T}({}^t e)$, so that E is an idempotent of $Z(\mathcal{O}G)$. Then*:
(a)

$$(\mathcal{O}G)E = (\mathcal{O}G)e(\mathcal{O}G), \quad e(\mathcal{O}G)e = e(\mathcal{O}G)Ee = (\mathcal{O}H)e$$

and the $\mathcal{O}$-linear map

$$f\colon\ \mathrm{Ind}_H^G((\mathcal{O}H)e) \to (\mathcal{O}G)E$$

such that $x \otimes_{\mathcal{O}H} b \otimes_{\mathcal{O}H} y \mapsto xby$ *for all* $x, y \in G$ *and all* $b \in (\mathcal{O}H)e$ *is an interior* G*-algebra isomorphism. Also the* $\mathcal{O}$*-linear map*

$$\phi\colon Z((\mathcal{O}H)e) \to Z\left(\mathrm{Ind}_H^G((\mathcal{O}H)e)\right)$$

such that $z \mapsto \sum_{t\in T}(t \otimes_{\mathcal{O}} z \otimes_{\mathcal{O}} t^{-1})$ *for all* $z \in Z((\mathcal{O}H)e)$ *is an* $\mathcal{O}$*-algebra isomorphism*;

(b) *The inclusion map* $\iota\colon (\mathcal{O}H)e \to (\mathcal{O}G)E$ *is an embedding of interior* H*-algebras*;

(c) *The functors*

$$\mathrm{Ind}_H^G(*) = (\mathcal{O}G)e \otimes_{(\mathcal{O}H)e} (*)\colon (\mathcal{O}H)e\text{-}mod \to (\mathcal{O}G)E\text{-}mod$$

and

$$e \cdot \mathrm{Res}^{\mathcal{O}G}_{\mathcal{O}H}(*) = e(\mathcal{O}G) \otimes_{(\mathcal{O}G)E} (*) : (\mathcal{O}G)E\text{-}mod \to (\mathcal{O}H)e\text{-}mod$$

exhibit a Morita equivalence between the Abelian categories $(\mathcal{O}H)e$*-mod and* $(\mathcal{O}G)E$*-mod with associated* $((\mathcal{O}H)e, (\mathcal{O}G)E)$*-bimodule* $e(\mathcal{O}G)$; *and*

(d) *Let* M *be an* $(\mathcal{O}H)e$*-module. Then*

$$\mathrm{Ind}_H^G(M) = (\mathcal{O}G)e \otimes_{(\mathcal{O}H)e} M = \bigoplus_{t\in T}(t \otimes_{\mathcal{O}} M)$$

and

$$\alpha(g \otimes_{(\mathcal{O}H)e} m) = \begin{cases} 0 & \text{if } g \notin H \\ 1 \otimes_{\mathcal{O}} (\alpha g)m & \\ & \text{if } g \in H, \text{ for all } \alpha \in (\mathcal{O}H)e, \text{ all } m \in M \text{ and all } g \in G. \end{cases}$$

Let e be an idempotent of $Z(\mathcal{O}H)$.

REMARK 1.3. Let $g \in G$. The following three conditions are equivalent:

(i) $e(\mathcal{O}(HgH))e = (0)$;

(ii) $e({}^g e) = 0$; and

(iii) $e(\mathcal{O}(HgH) \otimes_{\mathcal{O}H} V) = (0)$ for any module V in $(\mathcal{O}H)e$-mod.

Indeed, it is clear that (i) implies (ii) and (iii). Let $h_1, h_2 \in H$. Then $e(h_1 g h_2)e = h_1 e({}^g e) g h_2$, so that (ii) implies (i). Also if $V = (\mathcal{O}H)e$ in (iii), then

$$e\left(\mathcal{O}(HgH) \otimes_{\mathcal{O}H} (\mathcal{O}H)e\right) \cong e(\mathcal{O}(HgH)e)$$

in $(\mathcal{O}H)e$-mod and so (iii) implies (i).

Lemma 1.4 (E.C. Dade [4]). *Let $\mathcal{K}$ be a field and let e be an idempotent in $Z(\mathcal{K}H)$. Suppose that*

$$\dim\bigl(\mathrm{Hom}_{\mathcal{K}G}\bigl(\mathrm{Ind}_H^G(X), \mathrm{Ind}_H^G(Y)\bigr)\big/\mathcal{K}\bigr) = \dim(\mathrm{Hom}_{\mathcal{K}H}(X, Y)/\mathcal{K})$$

for any irreducible modules X, Y in $(\mathcal{K}H)e$-mod. Then $e({}^g e) = 0$ for all $g \in G - H$.

An immediate implication of Lemma 1.4 is:

Corollary 1.5. *Assume that $\mathcal{K}$ is a splitting field for G and H and that e is an idempotent of $Z(\mathcal{K}H)$ such that Ind_H^G defines an injective map $\mathrm{Ind}_H^G\colon \mathrm{Irr}_{\mathcal{K}}(e) \to \mathrm{Irr}_{\mathcal{K}}(G)$. Then $e({}^g e) = 0$ for all $g \in G - H$.*

For the remainder of this section, we assume that $(\mathcal{O}, \mathcal{K}, k)$ is a p-modular system that is "large enough" for all subgroups of G. As is standard, the natural ring epimorphism $-\colon \mathcal{O} \to k = \mathcal{O}/J(\mathcal{O})$ induces an epimorphism on all $\mathcal{O}$-algebras that is also denoted by $-$. Similarly for $\mathcal{O}$-modules.

Theorem 1.6 (cf. [5, V, Theorem 2.5], [9, Proposition 1] and [11, Theorem 1]). *Assume that $b \in Bl(\mathcal{O}H)$ is such that $b({}^g b) = 0$ for all $g \in G - H$ (as in* Proposition 1.2) *and let D be a defect group of b in H. Then*:
(a) Proposition 1.2 *applies (with $R = \mathcal{O}$), $B = \mathrm{Tr}_H^G(b) \in Bl(\mathcal{O}G)$ and D is a defect group of B in G;*
(b) *The functors* $\mathrm{Ind}_H^G(*) = (\mathcal{O}G) \otimes_{\mathcal{O}H} (*) = (\mathcal{O}G)b \otimes_{(\mathcal{O}H)b} (*)$:

$$(\mathcal{O}H)b\text{-}mod \to (\mathcal{O}G)B\text{-}mod \quad \textit{and} \quad b \cdot \mathrm{Res}_H^G(*)\colon (\mathcal{O}G)B\text{-}mod \to (\mathcal{O}H)b\text{-}mod$$

exhibit a Morita equivalence between the Abelian categories $(\mathcal{O}H)b$-mod and $(\mathcal{O}G)B$-mod. On the character level, this Morita equivalence induces the bijections:

$$\mathrm{Ind}_H^G\colon\ \mathrm{Irr}_{\mathcal{K}}(b) \to \mathrm{Irr}_{\mathcal{K}}(B), \quad \mathrm{Ind}_H^G\colon\ \mathrm{Irr}_k(b) \to \mathrm{Irr}_k(B)$$

and

$$\mathrm{Ind}_H^G\colon\ \mathrm{Irr}\, Br_{\mathcal{K}}(b) \to \mathrm{Irr}\, Br_{\mathcal{K}}(B).$$

Moreover, this Morita equivalence has associated bimodules:

$$(\mathcal{O}G)b \quad in \quad (\mathcal{O}G)B\text{-}mod\text{-}(\mathcal{O}H)b \quad and \quad b(\mathcal{O}G) \quad in \quad (\mathcal{O}H)b\text{-}mod\text{-}(\mathcal{O}G)B.$$

Here $(\mathcal{O}G)b$ when viewed as an $\mathcal{O}(G \times H)$-module is indecomposable with $\Delta D = \{(d, d) \mid d \in D\}$ and trivial ΔD-source and a similar fact holds for $b(\mathcal{O}G)$;

(c) *Let M be an indecomposable $(\mathcal{O}H)b$-module with vertex Q and Q-source V. Then* $\mathrm{Ind}_H^G(M) = \mathcal{O}G \otimes_{\mathcal{O}H} M = (\mathcal{O}G)b \otimes_{(\mathcal{O}H)b} M$ *in $(\mathcal{O}G)B$-mod is an indecomposable $(\mathcal{O}G)$-module with vertex Q and Q-source V*;
(d) *The above conditions hold over k for $\overline{b} \in Bl(kH)$ and $\overline{B} = \mathrm{Tr}_H^G\left(\overline{b}\right) \in Bl(kG)$, etc*;
(e) *The inclusion map $i\colon (\mathcal{O}H)b \to (\mathcal{O}G)B$ is an embedding of interior H-algebras so that i induces injective maps* ([10, Proposition 15.1])

$$i_*\colon \mathcal{PG}((\mathcal{O}H)b) \to \mathcal{PG}((\mathcal{O}G)B) \quad \text{and} \quad i_*\colon \mathcal{LPG}((\mathcal{O}H)b) \to \mathcal{LPG}((\mathcal{O}G)B).$$

Let D_γ be a defect pointed group of $(\mathcal{O}H)b$ as an H-algebra. Thus $i_*(D_\gamma) = D_{i(\gamma)}$, where $i(\gamma) = \left\{\gamma^{((\mathcal{O}G)E)^\times}\right\}$, is a defect pointed group of $(\mathcal{O}G)B$ as a G-algebra. Thus if $j \in \gamma$, then $j \in i(\gamma)$ and $j(\mathcal{O}G)Bj = jb(\mathcal{O}G)bj = j(\mathcal{O}H)bj$, so that these source algebras of b and B are equal as interior D-algebras; *and*
(f) *The Puig category of local pointed groups of b in $\mathcal{O}H$ and of B in $\mathcal{O}G$ are equivalent.*

The next result illuminates the hypothesis of [11, Theorem 1].

Proposition 1.7. *Let b be a block idempotent of $Z(\mathcal{O}H)$. The following four conditions are equivalent*:
(a) Ind_H^G *induces an injective map of* $\mathrm{Irr}_k\left(\overline{b}\right) \to \mathrm{Irr}_k(G)$;
(b) Ind_H^G *induces an injective map of* $\mathrm{Irr}_{\mathcal{K}}(b) \to \mathrm{Irr}_{\mathcal{K}}(G)$; *and*
(c) Ind_H^G *induces an injective map of* $\mathrm{Irr}\, Br_{\mathcal{K}}(b) \to \mathrm{Irr}\, Br_{\mathcal{K}}(G)$; *and*
(d) $b({}^g b) = 0$ *for all $g \in G - H$.*
In which case, Theorem 1.6 *applies so that $B = \mathrm{Tr}_H^G(b) \in Bl(\mathcal{O}G)$, the functor*

$$\mathrm{Ind}_H^G = (\mathcal{O}G)b \otimes_{(\mathcal{O}H)b} (*)\colon (\mathcal{O}H)b\text{-}mod \to (\mathcal{O}G)B\text{-}mod$$

induces a (*Morita*) *categorical equivalence*, *the maps of* (*a*), (*b*) *and* (*c*) *are bijections*, *etc*.

In our final result, (a), (b), (c) and (d) are presented in [9, Theorem 1] without proof. For the convenience of the reader, we shall include a proof of these items.

Theorem 1.8 (cf. [9, Theorem 1]). *Assume that $b \in Bl(\mathcal{O}H)$ is such that $b({}^g b) = 0$ for all $g \in G - H$* (*as in* Theorem 1.6). *Set $\Omega = \{{}^g b \mid g \in G\}$ so that $B = \left(\sum_{\omega\in\Omega}\omega\right) \in Bl(\mathcal{O}G)$, etc*.
(a) *Let $\left(P, \overline{b}_P\right)$ be a b-subpair of H. Then $\overline{b}_P\left({}^x\overline{b}_P\right) = 0$ for all $x \in C_G(P) - C_H(P)$,* Theorem 1.6 (d) *applies $s\left(\overline{b}_P\right) = \mathrm{Tr}_{C_H(P)}^{C_G(P)}\left(\overline{b}_P\right) \in Bl(kC_G(P))$, $\left(P, s\left(\overline{b}_P\right)\right)$ is a B-subpair of G and the k-linear map*

$$\begin{aligned}\mu\colon\ &\mathrm{Ind}_{C_H(P)}^{C_G(P)}\left(kC_H(P)\overline{b}_P\right) = kC_G(P) \otimes_{kC_H(P)} kC_H(P)\overline{b}_P \otimes_{kC_H(P)} kC_G(P)\\ &\to kC_G(P)s\left(\overline{b}_P\right)\end{aligned}$$

such that $x \otimes_{kC_H(P)} \alpha \otimes_{kC_H(P)} y \to x\alpha y$ *for all* $x, y \in C_G(P)$ *and all* $\alpha \in kC_H(P)\overline{b}_P$ *is an interior* $C_G(P)$*-algebra isomorphism. Also* $\mathrm{Ind}_H^G \colon \mathrm{Irr}_k(\overline{b}_P) \to \mathrm{Irr}_k(s(\overline{b}_P))$ *is a bijection*;

(b) *The map* $(P, \overline{b}_P) \mapsto (P, s(\overline{b}_P))$ *from the set of b-subpairs of* H *into the set of* B*-subpairs of* G *is injective*;

(c) *Let* $(Q, \overline{b_Q})$ *and* $(P, \overline{b_P})$ *be b-subpairs of* H. *Then*:

(i) $\{g \in G \mid {}^g(Q, s(\overline{b_Q})) = (P, s(\overline{b}_P))\} = C_G(P)\{h \in H \mid {}^h(Q, \overline{b_Q}) = (P, \overline{b}_P)\}$ *so that* $(Q, \overline{b_Q})$ *and* $(P, \overline{b_P})$ *are conjugate in* H *if and only if* $(Q, s(\overline{b_Q}))$ *and* $(P, s(\overline{b_P}))$ *are conjugate in* G, *and*

(ii) $(Q, \overline{b_Q}) \le (P, \overline{b}_P)$ *in* H *if and only if* $(Q, s(\overline{b_Q})) \le (P, s(\overline{b_P}))$ *in* G;

(d) *For any* B*-subpair* $(P', \overline{B}_{P'})$ *of* G *there is an* $x \in G$ *and a b-subpair* $(P, \overline{b}_P)$ *of* H *such that* ${}^x(P', \overline{B}_{P'}) = (P, s(\overline{b}_P))$; *consequently the Brauer category of* b *in* H *is equivalent to the Brauer category of* B *in* G;

(e) *Let* $(Q, \overline{b_Q})$ *be a b-subpair of* H. *The injective map* $i_* \colon \mathcal{LPG}((\mathcal{O}H)b) \to \mathcal{LPG}((\mathcal{O}G)B)$ *of* Theorem 1.6 *induces a bijection*

$$\begin{aligned} i_*^{(Q,\overline{b_Q})} \colon &\{Q_\gamma \in \mathcal{LPG}((\mathcal{O}H)b) \mid Q_\gamma \text{ is associated with } (Q, \overline{b_Q})\} \\ &\to \{Q_\delta \in \mathcal{LPG}((\mathcal{O}G)B) \mid Q_\delta \text{ is associated with } (Q, s(\overline{b_Q}))\} \end{aligned}$$

in which $Q_\gamma \mapsto Q_{i_*(\gamma)}$ *for all* $Q_\gamma \in \mathcal{LPG}((\mathcal{O}H)b)$ *such that* Q_γ *is associated with* $(Q, \overline{b_Q})$;

(f) *Let* $(P, \overline{b}_P)$ *be a b-subpair of* H *and let* $(P, s(\overline{b}_P))$ *be the corresponding* B*-subpair of* G. *Let* b_P *be the unique block idempotent of* $Z(\mathcal{O}C_H(P))$ *that "lifts"* $\overline{b}_P$. *Then* $b_P({}^x b_P) = 0$ *for all* $x \in C_G(P) - C_H(P)$, $s(b_P) = \mathrm{Tr}_{C_H(P)}^{C_G(P)}(b_P)$ *is a block idempotent of* $\mathcal{O}C_G(P)$ *that "lifts"* $s(\overline{b}_P)$ *and* Theorem 1.6 *applies to* $b_P \in Bl(\mathcal{O}C_H(P))$ *where* $C_H(P) \le C_G(P)$; *and*

(g) *Let* (D, b_D) *be a maximal b-subpair of* H. *Let* $P \le D$ *and let* $(P, \overline{b}_P)$ *be the unique b-subpair of* H *such that* $(P, \overline{b}_P) \le (D, \overline{b_D})$. *Then*

$$\mathrm{Ind}_{C_H(P)}^{C_G(P)}(*) \colon R_{\mathcal{K}}(C_H(P), b_P) \to R_{\mathcal{K}}(C_G(P), s(b_P))$$

is a perfect isometry and consequently induces the linear map

$$\mathrm{Ind}_{C_H(P)}^{C_G(P)}(*)_{p'} \colon CF_{p'}(C_H(P), b_P, \mathcal{K}) \to CF_{p'}(C_G(P), s(b_P), \mathcal{K}).$$

Let $u \in D$ *and set* $P = \langle \mu \rangle$. *Then*

$$d_G^{(u,s(b_P))} \circ \mathrm{Ind}_H^G(*) = \mathrm{Ind}_{C_H(P)}^{C_G(P)}(*)_{p'} \circ d_H^{(u,b_P)} \colon CF(H, b, \mathcal{K}) \to CF_{p'}(C_G(P), s(b_P), \mathcal{K}).$$

Consequently the perfect isometry $\mathrm{Ind}_H^G(*)$: $R_{\mathcal{K}}(H,b) \to R_{\mathcal{K}}(G,B)$ *is part of an isotopy between* b *and* B *with local system the family* $\{\mathrm{Ind}_{C_H(P)}^{C_G(P)}(*) \mid P \le D,\ P \text{ cyclic}\}$.

REMARK 1.9. In the situation of Theorem 1.8 and after Theorem 1.6 (a) has been established, the more general investigations of [6] apply (cf. [6, Remark 1.3 (a)]).

2. Preliminary results

Let G be a finite group and let $(\mathcal{O}, \mathcal{K}, k = \mathcal{O}/J(\mathcal{O}))$ be a p-modular system that is "large enough" for all subgroups of G. We shall, as in [3], set $CF_{p'}(G, \mathcal{K}) = \{ f \in CF(G, \mathcal{K}) \mid f(G - G_{p'}) = (0)\}$.

Let u be a p-element of G and set $P = \langle u \rangle$. Let $\chi \in \mathrm{Irr}_{\mathcal{K}}(G)$ and let $\phi \in \mathrm{Irr}\, Br_{\mathcal{K}}(C_G(P)) \subseteq CF_{p'}(C_G(P), \mathcal{K})$. We shall let $d_u(\chi, \phi)$ denote the generalized decomposition number associated to $u \in G_p$, $\chi \in \mathrm{Irr}_{\mathcal{K}}(G)$ and $\phi \in \mathrm{Irr}\, Br_{\mathcal{K}}(C_G(P))$, cf. [5, IV, Section 6]. Thus $d_G^u(\chi)(*) \in CF_{p'}(C_G(P), \mathcal{K})$ where $d_G^u(\chi)(s) = \chi(us) = \sum_{\phi \in \mathrm{Irr}\, Br_{\mathcal{K}}(C_G(P))} d_u(\chi, \phi)\phi(s)$ for all $s \in C_G(P)_{p'}$. Moreover, as in [3, Section 4A], if $b \in Bl(\mathcal{O}G)$ and $b_P \in Bl(\mathcal{O}C_G(P))$, then $d_G^{(u,b_P)}\colon CF(G, b, \mathcal{K}) \to CF_{p'}(C_G(P), b_P, \mathcal{K})$ is defined by: if $\alpha \in CF(G, b, \mathcal{K})$ and $s \in C_G(P)_{p'}$, then $\big(d_G^{(u,b_P)}(\alpha)\big)(s) = (b_P \cdot d_G^u(\alpha))(s) = \alpha(usb_P)$.

Since $\mathrm{Irr}_{\mathcal{K}}(b)$ is a basis of $CF(G, b, \mathcal{K})$, the $\mathcal{K}$-linear map $d_G^{(u,b_P)}$ is characterized by the well-known:

Lemma 2.1. *Let $\chi \in \mathrm{Irr}_{\mathcal{K}}(b)$. If $Br_P(b)\overline{b}_P = 0$, then $d_G^{(u,b_P)}(\chi) = 0$. If $Br_P(b)\overline{b}_P = \overline{b}_P$, then $d_G^{(u,b_P)}(\chi) = \sum_{\phi \in \mathrm{Irr}\, Br_{\mathcal{K}}(b_P)} d_u(\chi, \phi)\phi$*

Proof. With the notation and hypotheses of this lemma, the first statement is a consequence of Brauer's Second Main Theorem on Blocks ([5, IV, Theorem 6.1]) and the second statement is a consequence of [2, Theorem A2.1]. $\square$

REMARK 2.2. As above, if $\phi \in \mathrm{Irr}\, Br_{\mathcal{K}}(C_G(P))$ corresponds to $\gamma \in \mathcal{LP}((\mathcal{O}G)^P)$ (i.e., ϕ is the irreducible Brauer character obtained from the irreducible $kC_G(P)$-module $kC_G(P)Br_P(j)/J(kC_G(P)Br_P(j))$ for any $j \in \gamma$), then, by [10, Theorem 43.4] $d_u(\chi, \phi) = \chi(uj)$ for any $j \in \gamma$.

3. Proofs

As noted above, Lemma 1.1 and Proposition 1.2 are well-known and easy to prove.

Proof of Lemma 1.4. Assume the hypotheses of Lemma 1.4. Let S be a set of double (H, H)-coset representatives in G such that $1 \in S$ and let X, Y be irreducible modules in $(\mathcal{K}H)e$-mod. Here

$$\begin{aligned}\mathrm{Hom}_{\mathcal{K}G}\big(\mathrm{Ind}_H^G(X), \mathrm{Ind}_H^G(Y)\big) &\cong \mathrm{Hom}_{\mathcal{K}H}\left(X, \bigoplus_{s\in S}(\mathcal{K}(HsH) \otimes_{\mathcal{K}H} Y)\right)\\ &\cong \bigoplus_{s\in S} \mathrm{Hom}_{\mathcal{K}H}(X, \mathcal{K}(HsH) \otimes_{\mathcal{K}H} Y)\end{aligned}$$

in $\mathcal{K}$-mod. Thus $\mathrm{Hom}_{\mathcal{K}H}(X, \mathcal{K}(HsH) \otimes_{\mathcal{K}H} Y) = (0)$ for all $1 \neq s \in S$.

Fix $1 \neq s \in S$ and an irreducible module X in $(\mathcal{K}H)e$-mod.

We assert: $(*)$ $\mathrm{Hom}_{\mathcal{K}H}(X, \mathcal{K}(HsH) \otimes_{\mathcal{K}H} V) = (0)$ for all V in $(\mathcal{K}H)e$-mod.

Indeed, we may assume that V is reducible in $(\mathcal{K}H)e$-mod and we proceed by induction on $\dim(V/\mathcal{K})$. Let V_1 be a maximal submodule of V. Then

$$(0) \to V_1 \to V \to V/V_1 \to (0)$$

is a short exact sequence in $(\mathcal{K}H)e$-mod. Thus, since $\mathcal{K}(HsH)|(\mathcal{K}G)$ in $\mathcal{K}H$-mod-$\mathcal{K}H$,

$$(0) \to \mathcal{K}(HsH) \otimes_{\mathcal{K}H} V_1 \to \mathcal{K}(HsH) \otimes_{\mathcal{K}H} V \to \mathcal{K}(HsH) \otimes_{\mathcal{K}H} (V/V_1) \to (0)$$

is a short exact sequence in $\mathcal{K}H$-mod. Consequently

$$\begin{aligned}&\mathrm{Hom}_{\mathcal{K}H}(X, \mathcal{K}(HsH) \otimes_{\mathcal{K}H} V_1)\\ &\qquad \to \mathrm{Hom}_{\mathcal{K}H}(X, \mathcal{K}(HsH) \otimes_{\mathcal{K}H} V)\\ &\qquad\qquad \to \mathrm{Hom}_{\mathcal{K}H}(X, \mathcal{K}(HsH) \otimes_{\mathcal{K}H} (V/V_1))\end{aligned}$$

is exact in $\mathcal{K}$-mod and we conclude from the induction hypothesis that

$$\mathrm{Hom}_{\mathcal{K}H}(X, \mathcal{K}(HsH) \otimes_{\mathcal{K}H} V) = (0).$$

This establishes $(*)$.

Since X can be any irreducible $(\mathcal{K}H)e$-module, $(*)$ implies that $\mathrm{Soc}(e\mathcal{K}(HsH) \otimes_{\mathcal{K}H} V) = (0)$ for any module V in $(\mathcal{K}H)e$-mod. Thus $e\mathcal{K}(HsH)e \otimes_{\mathcal{K}H} V = (0)$ for any module V in $(\mathcal{K}H)e$-mod and we are done. □

Proof of Theorem 1.6. Assume the hypotheses of Theorem 1.6. Applying Proposition 1.2, [5, V, Lemma 1.2] and [5, III, Lemma 9.6], D is contained in a defect group of $B \in Bl(\mathcal{O}G)$. Since $\mathrm{Ind}_H^G\colon \mathrm{Irr}_{\mathcal{K}}(b) \to \mathrm{Irr}_{\mathcal{K}}(B)$ is bijective, [5, IV, Theorem 4.5] and degree considerations complete a proof of (a). Clearly $(\mathcal{O}G)b$ is indecomposable in $\mathcal{O}(G \times H)(B \otimes_{\mathcal{O}} b^0)$-mod and $D \times D$ is a defect group of $B \otimes_{\mathcal{O}} b^0 \in Bl(\mathcal{O}(G \times H))$. Also $(\mathcal{O}H)b|\mathrm{Res}^{G\times H}_{H\times H}((\mathcal{O}G)b)$ in $\mathcal{O}(H \times H)$-mod and $(\mathcal{O}H)b$ is indecomposable in $\mathcal{O}(H \times H)$-mod with ΔD as a vertex and trivial ΔD-source. Then [5, III, Lemma 4.6 (ii) and Corollary 6.8] implies the last part of (b). Thus (b) holds, [5, III, Corollary 4.7] yields (c) and (d) and (e) are clear. Finally (e) and [5, Theorem 47.10 (b)] yield (f). □

Proof of Proposition 1.7. Assume the situation of this proposition. Let V and W be irreducible $(kH)\overline{b}$-modules with irreducible characters ϕ_V, ϕ_W in $\mathrm{Irr}_k(\overline{b})$ and irreducible Brauer characters β_V, β_W in $\mathrm{Irr}\, Br_{\mathcal{K}}(b)$.

Assume that (a) holds. Then $\mathrm{Ind}_H^G(V)$ is an irreducible kG-module and $\mathrm{Ind}_H^G(\beta_V) = \beta_{\mathrm{Ind}_H^G(V)} \in \mathrm{Irr}\, Br_{\mathcal{K}}(G)$. Similarly $\mathrm{Ind}_H^G(W)$ is an irreducible kG-module and $\mathrm{Ind}_H^G(\beta_W) = \beta_{\mathrm{Ind}_H^G(V)} \in \mathrm{Irr}\, Br_{\mathcal{K}}(G)$. Suppose that $\beta_{\mathrm{Ind}_H^G(V)} = \beta_{\mathrm{Ind}_H^G(W)}$. Then

$$\phi_{\mathrm{Ind}_H^G(V)} = \overline{\beta_{\mathrm{Ind}_H^G(V)}} = \overline{\beta_{\mathrm{Ind}_H^G(W)}} = \phi_{\mathrm{Ind}_H^G(W)}, \quad \mathrm{Ind}_H^G(V) \cong \mathrm{Ind}_H^G(W) \quad \text{in} \quad kG\text{-mod}$$

and hence $\mathrm{Ind}_H^G(\phi_V) = \mathrm{Ind}_H^G(\phi_W)$. But then $\phi_V = \phi_W$, $V \cong W$ in $(kH)\overline{b}$-mod and $\beta_V = \beta_W$, so that (c) follows.

Assume that (c) holds. Then $\overline{\beta}_V = \phi_V \in \mathrm{Irr}_k\left(\overline{b}\right)$ and $\mathrm{Ind}_H^G(\beta_V) = \beta_{\mathrm{Ind}_H^G(V)} \in \mathrm{Irr}\, Br_{\mathcal{K}}(G)$. Thus $\mathrm{Ind}_H^G(\phi_V) = \phi_{\mathrm{Ind}_H^G(V)} \in \mathrm{Irr}_{\mathcal{K}}(G)$. Similarly $\beta_{\mathrm{Ind}_H^G(W)} \in \mathrm{Irr}\, Br_{\mathcal{K}}(G)$ and $\mathrm{Ind}_H^G(\phi_W) = \phi_{\mathrm{Ind}_H^G(W)} \in \mathrm{Irr}_k(G)$. Suppose that $\mathrm{Ind}_H^G(\phi_V) = \mathrm{Ind}_H^G(\phi_W)$. Then $\beta_{\mathrm{Ind}_H^G(V)} = \beta_{\mathrm{Ind}_H^G(W)} = \mathrm{Ind}_H^G(\beta_V) = \mathrm{Ind}_H^G(\beta_W)$, so that $\beta_V = \beta_W$, $\phi_V = \phi_W$ and (a) holds. Consequently (a) and (c) are equivalent.

That (d) implies (a), (b) and (c) is a consequence of Theorem 1.6 (b). Assume (a) and let $g \in G - H$. Then $M = b\mathcal{O}(HgH)b$ is an $\mathcal{O}$-lattice where $\overline{M} = \overline{b}k(HgH)\overline{b} = (0)$ by Corollary 1.5. Consequently $b\mathcal{O}(HgH)b = (0)$ and (d) holds.

Assume (b) and for each $\chi \in \mathrm{Irr}_{\mathcal{K}}(b)$, let $e_\chi = (\chi(1)/|H|)\left(\sum_{h\in H}(\chi(h^{-1})h)\right)$ be the primitive idempotent of $Z(\mathcal{K}H)$ corresponding to χ. Then $e_\chi({}^g e_\chi) = 0$ for all $g \in G - H$ by Corollary 1.5 and $\mathrm{Tr}_H^G(e_\chi) = e_{\mathrm{Ind}_H^G(\chi)}$ is the primitive idempotent of $Z(\mathcal{K}G)$ corresponding to $\mathrm{Ind}_H^G(\chi)$. Let $\chi, \psi \in \mathrm{Irr}_{\mathcal{K}}(b)$ and let $g \in G - H$. Then $e_\chi({}^g e_\psi) = \left(e_\chi \,\mathrm{Tr}_H^G(e_\chi)\right)\left(\mathrm{Tr}_H^G(e_\chi)^g e_\psi\right) = 0$, (d) holds and our proof is complete. $\square$

Proof of Theorem 1.8. For (a), note that $Br_P(b)\overline{b}_P = \overline{b}_P$. Let $x \in C_G(P) - C_H(P)$; then

$$\overline{b}_P\left({}^x\overline{b}_P\right) = \overline{b}_P\, Br_P(b)\, Br_P({}^x b)\left({}^x\overline{b}_P\right) = \overline{b}_P\, Br_P(b({}^x b))\left({}^x\overline{b}_P\right) = 0.$$

Thus $\mathrm{Stab}_{C_G(P)}\left(\overline{b}_P\right) = C_H(P)$ and, since $Br_P(B)\overline{b}_P = \overline{b}_P$, we conclude that

$$Br_P(B)\, \mathrm{Tr}_{C_H(P)}^{C_G(P)}\left(\overline{b}_P\right) = \mathrm{Tr}_{C_H(P)}^{C_G(P)}\left(\overline{b}_P\right).$$

Then Proposition 1.2 and Theorem 1.6 yield (a). Since $Br_P(b)s\left(\overline{b}_P\right) = \overline{b}_P$, (b) holds.

Let $\left(Q, \overline{b}_Q\right)$ and $\left(P, \overline{b}_P\right)$ be b-subpairs of H and let S be a left transversal of $C_H(Q)$ in $C_G(Q)$ with $1 \in S$, so that $C_G(Q) = \bigcup_{s\in S} sC_H(Q)$ is disjoint. Let $h \in H$ be such that ${}^h\left(Q, \overline{b}_Q\right) = \left(P, \overline{b}_P\right)$. Then

$${}^h s\left(\overline{b}_Q\right) = \sum_{s\in S} {}^{(hs)}\overline{b}_Q = \sum_{s\in S} {}^{(hsh^{-1})}\left({}^h\overline{b}_Q\right) = s\left(\overline{b}_P\right)$$

and hence

$$C_G(P)\left\{h \in H \;\middle|\; {}^h\left(Q, \overline{b}_Q\right) = \left(P, \overline{b}_P\right)\right\} \leq \left\{g \in G \;\middle|\; {}^g\left(Q, s\left(\overline{b}_Q\right)\right) = \left(P, s\left(\overline{b}_P\right)\right)\right\}.$$

Conversely, let $g \in G$ be such that ${}^g\big(Q, s(\overline{b}_Q)\big) = \big(P, s(\overline{b}_P)\big)$. Then ${}^gB = B$ and ${}^g\Omega = \Omega$. Let U be a left transversal of $C_H(P)$ in $C_G(P)$ with $1 \in U$, so that $C_G(P) = \bigcup_{u\in U} uC_H(P)$ is disjoint. Here ${}^gs(\overline{b}_Q) = s(\overline{b}_P) = \sum_{u\in U} Br_P({}^ub)({}^u\overline{b}_P)$, $Br_Q(b)s(\overline{b}_Q) = \overline{b}_Q$ and $Br_P({}^ub)({}^u\overline{b}_P) = {}^u\overline{b}_P$ for all $u \in U$. Thus

$$0 \neq {}^g\overline{b}_Q = Br_P({}^gb)s(\overline{b}_P) = \sum_{u\in U} Br_P({}^gb)({}^u\overline{b}_P).$$

We conclude that ${}^gb = {}^bu$ for some $u \in U$ and so $g = uh$ for some $h \in H$. But then ${}^g\big(Q, s(\overline{b}_Q)\big) = {}^u\big({}^hQ, {}^hs(\overline{b}_Q)\big) = \big(P, s(\overline{b}_P)\big)$ and $\big({}^hQ, {}^hs(\overline{b}_Q)\big) = \big(P, s(\overline{b}_P)\big)$. Since $Br_Q(b)s(\overline{b}_Q) = \overline{b}_Q$, we have $Br_P(b)s(\overline{b}_P) = {}^h\overline{b}_Q$ and then ${}^h\overline{b}_Q = \overline{b}_P$, which completes a proof of (c) (i).

For a proof of (c) (ii), it suffices to assume that $Q \trianglelefteq P$. First suppose that $\big(Q, \overline{b}_Q\big) \leq \big(P, \overline{b}_P\big)$. Thus $\overline{b}_Q$ is P-stable and $Br_P(b_Q)\overline{b}_P = \overline{b}_P$. As $P \leq N_G(Q)$, we conclude that P stabilizes $s(\overline{b}_Q)$. Let U be a left transversal of $C_H(P)$ in $C_G(P)$ with $1 \in U$. Here

$$Br_P\big(s(\overline{b}_Q)\big)\overline{b}_P = Br_P\big(s(\overline{b}_Q)\big)\, Br_P\big(\overline{b}_Q\big)\overline{b}_P = Br_P\big(\overline{b}_Q\big)\overline{b}_P = \overline{b}_P$$

and, since $C_G(P) \leq C_G(Q)$, we have $Br_P\big(s(\overline{b}_Q)\big)^u\overline{b}_P = {}^u\overline{b}_P$ for all $u \in U$. Thus $Br_P\big(s(\overline{b}_Q)\big)s(\overline{b}_P) = s(\overline{b}_P)$. Conversely, suppose that $\big(Q, s(\overline{b}_Q)\big) \leq \big(P, s(\overline{b}_P)\big)$. Then $s(\overline{b}_Q) \in (kC_G(Q))^P$ and $Br_P\big(s(\overline{b}_Q)\big)s(\overline{b}_P) = s(\overline{b}_P)$. Utilizing [10, Lemma 40.2],

$$\begin{aligned} s(\overline{b}_P)\, Br_P(b) = \overline{b}_P &= Br_P\big(s(\overline{b}_Q)\big)s(\overline{b}_P)\, Br_P(b) \\ &= Br_{P/Q}(s(b_Q)\, Br_Q(b))\overline{b}_P = Br_{P/Q}\big(\overline{b}_Q\big)\overline{b}_P = Br_P\big(\overline{b}_Q\big)\overline{b}_P. \end{aligned}$$

Since $s(\overline{b}_Q)\, Br_Q(b) = \overline{b}_Q$, $\overline{b}_Q$ is P-stable and so $\big(Q, \overline{b}_Q\big) \leq \big(P, \overline{b}_P\big)$ which completes a proof of (c) (ii).

Let $\big(P', \overline{B}_{P'}\big)$ be a B-subpair of G. Let (D, b_D) be a maximal b-subpair of H; thus $\big(D, s(\overline{b}_D)\big)$ is a maximal B-subpair of G. Then there is an $x \in G$ such that ${}^x\big(P', \overline{B}_{P'}\big) \leq \big(D, s(\overline{b}_D)\big)$. Thus $\big({}^xP', {}^x\overline{B}_{P'}\big) \leq \big(D, s(\overline{b}_D)\big)$ and setting $Q = {}^xP'$, we have $\big(Q, \overline{b}_Q\big) \leq \big(D, \overline{b}_D\big)$ for a unique $\overline{b}_Q \in Bl(kC_H(Q))$. But then $\big(Q, s(\overline{b}_Q)\big) \leq \big(D, s(\overline{b}_D)\big)$; consequently ${}^x\overline{B}_{P'} = s(\overline{b}_Q)$ and ${}^x\big(P', \overline{B}_{P'}\big) = \big(Q, s(\overline{b}_Q)\big)$, which completes a proof of (d).

For (e), let $\big(Q, \overline{b}_Q\big)$ be a b-subpair of H. By (a), $kC_H(Q)\overline{b}_Q$-mod and $kC_G(Q)\overline{B}_Q$-mod are Morita equivalent. Thus $\big|\mathcal{P}\big(kC_H(Q)\overline{b}_Q\big)\big| = \big|\mathcal{P}\big(kC_G(Q)\overline{B}_Q\big)\big|$. Clearly

$$\Big|\Big\{Q_\gamma \in \mathcal{LPG}((\mathcal{O}H)b) \;\Big|\; Q_\gamma \text{ is associated with } \big(Q, \overline{b}_Q\big)\Big\}\Big| = \big|\mathcal{P}\big(kC_H(Q)\overline{b}_Q\big)\big|$$

and

$$\Big|\Big\{Q_\delta \in \mathcal{LPG}((\mathcal{O}G)B) \;\Big|\; Q_\delta \text{ is associated with } \big(Q, s(\overline{b}_Q)\big)\Big\}\Big| = \big|\mathcal{P}\big(kC_G(Q)s(\overline{b}_Q)\big)\big|.$$

Also if $Q_\gamma \in \mathcal{LPG}((\mathcal{O}H)b)$ and Q_γ is associated with $(Q, \overline{b}_Q)$ and $j \in \gamma$, then

$$Br_Q(j)\overline{b}_Q = Br_Q(j) = Br_Q(j)\overline{b}_Q s(\overline{b}_Q) = Br_Q(j)s(\overline{b}_Q).$$

Thus $\iota_*(Q_\gamma) \in \mathcal{LPG}((\mathcal{O}G)B)$ and $i_*(Q_\gamma)$ is associated with $(Q, s(\overline{b}_Q))$. The desired conclusion now follows from Theorem 1.6 (e).

Let $(P,\overline{b}_P)$, $(P,s(\overline{b}_P))$ and b_P be as in (f). Note that Ind_H^G: $\mathrm{Irr}_k(\overline{b}_P) \to \mathrm{Irr}_k(s(\overline{b}_P))$ is a bijection by (a). Then Proposition 1.2 and Theorem 1.6 yield (f).

Let $(D, \overline{b}_D)$ and (P, b_P) be as in (g). Then Theorem 1.6 (b) and [3, Proposition 1.2] imply that $\mathrm{Ind}_{C_H(P)}^{C_G(P)}(*)$: $\mathcal{R}_{\mathcal{K}}(C_H(P), b_P) \to \mathcal{R}_{\mathcal{K}}(C_G(P), s(b_P))$ is a perfect isometry that induces the linear map

$$\mathrm{Ind}_{C_H(P)}^{C_G(P)}(*)_{p'}:\ CF_{p'}(C_H(P), b_P, \mathcal{K}) \to CF_{p'}(C_G(P), s(b_P), \mathcal{K}).$$

Let $u \in D$, set $P = \langle u\rangle$ and let $\psi \in \mathrm{Irr}_{\mathcal{K}}(b)$. Then, by Lemma 2.1,

$$\mathrm{Ind}_{C_H(P)}^{C_G(P)}(d^H(u, b_P)(\psi)) = \sum_{\phi\in \mathrm{Irr}\, Br_{\mathcal{K}}(b_P)} d_u(\psi, \phi)\big(\mathrm{Ind}_{C_H(P)}^{C_G(P)}(\phi)\big)$$

and

$$d_G^{(u,s(b_P))}\big(\mathrm{Ind}_H^G(\psi)\big) = \sum_{\phi\in \mathrm{Irr}\, Br_{\mathcal{K}}(b_P)} \Big(d_u\big(\mathrm{Ind}_H^G(\psi), \mathrm{Ind}_{C_H(P)}^{C_G(P)}(\phi)\big)\, \mathrm{Ind}_{C_H(P)}^{C_G(P)}(\phi)\Big).$$

The desired conclusion now follows from [11, Theorem 1 (iv)]. An alternate proof can be obtained from [10, Theorem 43.4]. Indeed, let $\phi \in \mathrm{Irr}\, Br_{\mathcal{K}}(b_P)$ and let $\gamma \in \mathcal{LP}(((\mathcal{O}H)b)^P)$ correspond as in Remark 2.2. It is easy to see that $\mathrm{Ind}_{C_H(P)}^{C_G(P)}(\phi) \in \mathrm{Irr}\, Br_{\mathcal{K}}(s(b_P))$ corresponds $i(\gamma) \in \mathcal{L}P((\mathcal{O}G)B)^P)$. Let $j \in \gamma$. Here Proposition 1.2 (d) implies that $\mathrm{Ind}_H^G(\psi)(uj) = \psi(uj)$ and the desired conclusion follows from Remark 2.2. □

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